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Is MFM really useful?

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Abstract. The ill-posed linear inverse problems, characterised by Fredholm integral equations of the first kind, are encountered in many areas of science and technology. This type of problems present some loss of information under the inversion process. The loss of information often makes the inversion process very difficult. Magnetic force microscopy (MFM) is a technique where problems related to loss of information occur. Work is presented here to understand what can be measured by the magnetic force microscope. A simple model is constructed, where the magnetic tip is approximated by a point dipole. Given the force $F(\mathbf{r})$ acting on the dipole tip, we attempt to determine the magnetization distributlon in a thin ferromagnetic film, $M(\mathbf{r})$. This calculation should be interesting due to the rapidiy growing interest in magnetic thin films and magnetic multilayers.

PACS. 68.37.Rt Magnetic force microscopy (MFM) – 02.30.Rz Integral equations – 02.30.Zz Inverse problems

1 Introduction

The remarkable successes of the scanning tunneling microscope [1] and the atomic force microscope [2] has led to the idea of achieving a similar resolution in magnetic structures. The magnetic force microscope (MFM), a variant of the atomic force microscope, images surface magnetic features on a sub-100 nm scale without the need for involved sample preparation [3].

Preliminary works were mainly devoted to the imaging of written bit structures in several magnetic recording materials [4–6] and to the understanding of contrast mechanism in the MFM [7–9]. Moreover, magnetic domain walls in thin Permalloy films [10], iron single crystals [11] and magnetite [12] have been observed successfully.

The progress made in the last decade has turned the MFM from a research tool into one of the most widely used magnetic imaging techniques. Its ease of use and high resolution is striking, but the only drawback seems to be the interpretation of MFM images correctly and quantitatively. Recent advances like the tip perturbation work [22,28], the dissipation imaging [13], and the domain contrast work [14] have to some extent ameliorated the ability to interpret MFM images. Moreover, works which involve applying fields during MFM experiments have given significant information about the micromagnetics of the sample [15,16]. The MFM has also been used to study magnetostrictive effects at submicron scales [17].

The heart of the MFM is the magnetic tip attached to a cantilever. Due to the interaction between the tip and sample, the cantilever is deflected. This mechanical response of the cantilever is used to sense the interactions between the tip and sample as a function of the tip position. In experiments, both AC and DC detection techniques are employed. DC detection monitors the DC deflection of the cantilever which depends on the force between the tip and sample. AC detection techniques are sensitive to the resonant frequency of the cantilever. Therefore, it is related to force gradient between the tip and sample.

Note that both the force and force gradient are related to the magnetic field or spatial derivatives of the magnetic field from the sample. The effect of the magnetic field due to the sample on the tip is in principle not sufficient for obtaining the magnetization of the sample. This is because there may be an infinite number of magnetization patterns leading to the same magnetic field. In other words, the determination of true magnetization distribution of the sample from MFM force data is in principle not possible [18]. Under best conditions, only a map of the magnetic charge density, in bulk samples $-\nabla \cdot \mathbf{M}$ and at surfaces $\mathbf{M} \cdot \mathbf{n}$ where $\mathbf{n}$ denotes the surface normal, can be obtained. One cannot recover the magnetization from this quantity as well. On the other hand, this obstacle has not stopped several attempts which aim to recover magnetization distribution of the sample [19,20].

In practice, most MFM images are more complicated than one might have thought due to the perturbative interactions between the tip and sample. The localized magnetic field produced by the tip can affect the magnetic structures under investigation and *vice versa* [21]. The strength of these mutual perturbations depends on the tip and the sample materials, geometry and imaging conditions. For example, in the case of permanent magnet

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samples, the tip stray field does not modify the magnetic state of the sample. However, strong magnetic fields, of the order of 1 tesla, may be experienced by the tip in close proximity to the sample. The perturbations lead to several effects like magnetic dissipation, domain contrast and magnetization reversal [23,24]. Although these perturbations were initially regarded as unwanted effects, it has been found that they give information about the localized susceptibility of the sample [22,28].

The resolution of the instrument depends on several parameters like the minimum detectable force, the tip shape, and scanning height. For example, the effect of tip shape and scanning height on magnetic domain images was investigated in [25]. In the simplest case, neglecting the real geometry, the magnetic tip can be regarded as a point probe. One can then quantitatively determine the effective magnetic dipole and monopole moments of the tip as well as their imaginary location within the real tip [26]. From the analysis in [27], it is clear that the point-monopole and dipole models do not accurately describe the imaging mechanism in the MFM.

Most of the theory which has been developed to explain the MFM experiments relies on the assumption that both the tip and sample have a static magnetization state during MFM operation [28]. There have been some exceptions where the tip or sample magnetizations have been allowed to rotate. For example, in [29], the tip magnetization was permitted to rotate, whereas in [30] the magnetization of a low anisotropy sample was allowed to rotate.

Most theoretical works start with a simple model involving a magnetic monopolar or dipolar tip that interacts non-perturbatively with the sample. This model is useful for basic image interpretation but in practice is inevitably insufficient for a quantitative interpretation of MFM images [6,27]. In reality, the tip structure is both geometrically and magnetically complicated. Therefore, it is better to go beyond the point probe approximation.

Given the tip shape, the MFM response can be calculated by performing an integration over the tip volume. A number of MFM models that consider a particular tip shape have been setup with a wide variety of tip-sample geometries. In these models, some geometries were chosen to obtain convenient integration over the tip volume, while others were chosen to represent a real tip-sample configuration. For example, in [7], the simplest configuration can be found where both sample and the tip represent a magnetic dipole. Contrary to this simple case, [31] studies a perpendicular recording media using a cylindrical tip. In another work, a more realistic tip, modeled by a truncated pyramid, is used to probe a sample with periodic domains and perpendicular anisotropy [32].

More elegantly, in the absence of perturbations, one can calculate the Green's function of the tip by imaging a point-like sample [33–35]. Other computational techniques that have proved useful are reciprocity [36] and Fourier integrals [37].

The plan of the letter is as follows: In the next section, the problem of magnetization reconstruction is formulated for the general case. In the third section, we will focus on a restricted problem of determining the magnetization distribution of a thin ferromagnetic film. Then, we will define the smeared magnetization and the ambiguity in the smeared magnetization. Finally, we will present some conclusions.

2 General case

Neglecting the real geometries of the tip and sample and considering static magnetizations, we will now derive a relation between the force and the sample's magnetization distribution. We also assume that the sample magnetization is not perturbed by the stray field of the tip and *vice versa*. The interaction will be attributed to magnetic dipolar interaction between the magnetic field caused by the sample and the magnetization of the tip. Therefore, the interaction energy is of the form

$$E(\mathbf{r}') = -\int_t \mathbf{m}(\mathbf{r}') \cdot \mathbf{B}(\mathbf{r}') \mathrm{d}V' \quad (1)$$

where $\mathbf{B}(\mathbf{r}')$ denotes the magnetic field at $\mathbf{r}'$ caused by the sample's magnetization distribution and $\mathbf{m}(\mathbf{r}')$ denotes the magnetization of the tip. If we denote the magnetization distribution of the sample by $\mathbf{M}(\mathbf{r}'')$, the magnetic field generated by the sample will be as follows:

$$\mathbf{B}(\mathbf{r}') = \int_s [\nabla'' \times [\mathbf{M}(\mathbf{r}'') \times \nabla'']] \frac{1}{|\mathbf{r} - \mathbf{r}''|} \mathrm{d}V'' \quad (2)$$

which is taken over the entire sample volume. We stress that in noncontact scanning force microscopy the tip does *not* contact the surface under investigation. If we expand the triple vector product in (2), we encounter a term containing the delta function $\delta(\mathbf{r}' - \mathbf{r}'')$, which implies a contact between the sample and the tip. Due to the experimental restriction, this term will be neglected in the subsequent calculations. Consequently, the energy of the interaction becomes

$$E(\mathbf{r}') = \int_s \nabla_i'' \nabla_j'' M_j(\mathbf{r}'') \frac{1}{|\mathbf{r} - \mathbf{r}''|} \mathrm{d}V'' \int_t m_i(\mathbf{r}') \mathrm{d}V'. \quad (3)$$

The force exerted on the tip can be readily obtained by the definition $\mathbf{F} = -\nabla E$. The kth component of the force is then given by the expression:

$$F_k(\mathbf{r}') = \int_s \nabla_i'' \nabla_j'' \nabla_k'' M_j(\mathbf{r}'') \frac{1}{|\mathbf{r} - \mathbf{r}''|} \mathrm{d}V'' \int_t m_i(\mathbf{r}') \mathrm{d}V'. \quad (4)$$

As the simplest magnetic tip one can imagine is a point dipole, we will approximate the tip by a point dipole. If we denote the position vector of the dipole tip by $\mathbf{R}$, then (4) becomes

$$F_k(\mathbf{R}) = \int_s \left[\mu_i \nabla_i \nabla_j \nabla_k \frac{1}{|\mathbf{R} - \mathbf{r}''|}\right] M_j(\mathbf{r}'') \mathrm{d}V'' \quad (5)$$

$$= K_{jk}(\mathbf{R} - \mathbf{r}'') M_j(\mathbf{r}'') \mathrm{d}V'' \quad (6)$$

where $\mu_i = \int_t m_i \mathrm{d}V'$ is the magnetic moment of the tip. It is clear that (6) is a Fredholm integral equation for the sample's magnetization distribution $M_j(\mathbf{r}'')$ in terms of the force on the dipole tip $F_k(\mathbf{R})$, where the term $K_{jk}(\mathbf{R}-\mathbf{r}'')$ is the *kernel* of this integral equation.

3 Thin ferromagnetic film

We will now focus our attention on a restricted problem. That is, we try to determine the magnetization distribution of a ferromagnetic thin film residing in the $z = 0$ plane, where we represent the magnetization distribution by

$$\mathbf{M}(\mathbf{r}) = \mathbf{M}(x,y)\delta(z). \tag{7}$$

Moreover we assume that we measure the force on the tip over a plane with fixed separation with respect to the film. For this reason, it is convenient to work with z and the Fourier transform variables in the xy plane, namely $\mathbf{q}'' = (q^x, q^y)$. Applying convolution theorem to (6) we obtain the following expression

$$\widetilde{F}_i\left(\mathbf{q}^{\|}, q^z\right) = \left(\sqrt{2\pi}\right)^3 \widetilde{K}_{ij}(\mathbf{q})\widetilde{M}_j(\mathbf{q}). \tag{8}$$

We can now Fourier transform this equation with respect to the variable q^z

$$F_i\left(\mathbf{q}^{\|}, z\right) = 2\pi \int \mathrm{e}^{\mathrm{i}q^z z}\widetilde{K}_{ij}(\mathbf{q})\widetilde{M}_j(\mathbf{q})\mathrm{d}q^z \tag{9}$$

where the Fourier transformation of the kernel $\widetilde{K}_{ij}(\mathbf{q})$ is given by

$$\widetilde{K}_{ij}(\mathbf{q}) = -\frac{4\pi\mathrm{i}}{q^2} q_i q_j q_k \mu_k \tag{10}$$

Fourier transforming the magnetization of the ferromagnetic thin film gives $\widetilde{\mathbf{M}}(\mathbf{q}) = c\widetilde{\mathbf{M}}(\mathbf{q}'')$, where c is a constant due to the fact that the Fourier transformation of a delta function is a constant. Using the last two equation in (9) and performing the integral over q^z we find the following result for a *finite* tip-sample separation ($z \neq 0$):

$$F_i\left(\mathbf{q}^{\|}, z\right) = -8\pi^3\mathrm{i}\theta \frac{\mathrm{e}^{-q^{\|}z}}{q^{\|}} q_i^+ q_j^+ \widetilde{M}_j\left(\mathbf{q}^{\|}\right) \tag{11}$$

where $\theta = \mu \cdot \mathbf{q}^+$ is a scalar quantity and $\mathbf{q}^+ = \mathbf{q}^{\|} + \mathrm{i}q^{\|}\hat{\mathbf{z}}$ is a complex vector. For convenience we may rewrite (11) as follows:

$$F_i\left(\mathbf{q}^{\|}, z\right) = -8\pi^3\mathrm{i}\theta \frac{\mathrm{e}^{-q^{\|}z}}{q^{\|}} \widetilde{K}_{ij}^{\|}\left(\mathbf{q}^{\|}\right) \widetilde{M}_j\left(\mathbf{q}^{\|}\right) \tag{12}$$

where the Fourier transformed kernel is defined by $\widetilde{K}_{ij}^{\|}(\mathbf{q}^{\|}) = q_i^+ q_j^+$.

As one can see from (12), we may determine the magnetization distribution of the film, if we can invert the Fourier transformed kernel $\widetilde{K}_{ij}^{\|}(\mathbf{q}^{\|})$. We now prove that the inverse of the kernel only exists in a one-dimensional subspace. To demonstrate that this is the case, we should first determine the eigenvalues and eigenvectors of the kernel. The eigenvalues of the kernel can be found from the secular equation. We find that all three eigenvalues are equal to zero:

$$\lambda_1 = \lambda_2 = \lambda_3 = 0. \tag{13}$$

From this result, it might be thought that the kernel was completely non-invertible. However, this is not the case as a finite force on the tip is measured in experiments. The reason for the invertibility is that the kernel is a *non-Hermitian* matrix, which is constructed by the product of two complex vectors.

We will also prove that not all eigenvectors of the kernel exist. This property allows a partial inverse be defined in the relevant subspace. We may quickly check that $\mathbf{q}^+$ and $\mathbf{q}^+ \times (\mathbf{q}^+)^* = \mathbf{Q}$ are eigenvectors with zero eigenvalue as $\mathbf{q}^+ \cdot \mathbf{q}^+ = 0 = \mathbf{q}^+ \cdot \mathbf{Q}$, where * shows complex conjugate. However, the third linearly independent vector, $\mathbf{q}^- = (\mathbf{q}^+)^*$, is not an eigenvector of the kernel. Thus, one eigenvector of the kernel is absent.

Let us now construct the partial inverse in the subspace spanned by the vector $\mathbf{q}^-$, say $\widetilde{\mathcal{K}}_{ij}^{-1}$:

$$\widetilde{\mathcal{K}}_{ij}^{-1} = \frac{q_i^- q_j^-}{4{q^{\|}}^4} \cdot \tag{14}$$

Note that if we multiply the kernel from the left side by its partial inverse we get $\widetilde{\mathcal{K}}_{ij}^{-1}\widetilde{K}_{jk}^{\|} = q_i^- q_k^+ / 2{q^{\|}}^2$, where we use the relation $q_j^- q_j^+ = 2{q^{\|}}^2$. This is the identity matrix in the subspace spanned by q_i^-, as we have the relation:

$$\left(\frac{q_i^- q_k^+}{2{q^{\|}}^2}\right) q_k^- = q_i^- \tag{15}$$

(or q_i^+ if we use right multiplication). This is precisely the subspace associated with the non-existent eigenvector.

4 Smeared magnetization

Having constructed the partial inverse, we now turn to the force expression in (12). This can be rearranged as follows:

$$-\frac{{q^{\|}}^2}{8\mathrm{i}\pi^3\theta}\widetilde{\mathcal{K}}_{ki}^{-1}\left(\mathbf{q}^{\|}\right) F_i\left(\mathbf{q}^{\|}, z\right) = q^{\|}\mathrm{e}^{-q^{\|}z}\widetilde{M}_k\left(\mathbf{q}^{\|}\right) \tag{16}$$

where we have multiplied both sides by the factor $q^{\|}$ to get a simple form for the inverse kernel. We stress that the exponential factor $\mathrm{e}^{-q^{\|}z}$ is problematic. This cannot be transferred to the left hand side of the equation, as it could lead to a strongly divergent inverse Fourier transform. The physical significance of this is that the force on the tip is insensitive to details of magnetization on length scales less than the separation between the film and the tip.

We must now reconcile ourselves to recovering the partial information contained in the *smeared* magnetization which is defined as follows

$$\widetilde{M}_i^s\left(\mathbf{q}^{\|}, z\right) = q^{\|} \mathrm{e}^{-q^{\|} z} \widetilde{M}_i\left(\mathbf{q}^{\|}\right) \tag{17}$$

which when Fourier transformed gives the result:

$$M_i^s\left(\mathbf{r}^{\|}, z\right) = \int \left[\frac{3z^2}{\left(\left|\mathbf{r}^{\|} - \mathbf{r}^{\|\prime}\right|^2 + z^2\right)^{5/2}} - \frac{1}{\left(\left|\mathbf{r}^{\|} - \mathbf{r}^{\|\prime}\right|^2 + z^2\right)^{3/2}} \right] M_i\left(\mathbf{r}^{\|\prime}\right) \mathrm{d}\mathbf{r}^{\|\prime}. \tag{18}$$

Returning to the smeared magnetization expression, we may rewrite the right hand side as $M_k^s(\mathbf{q}^{\|}, z)$ and perform the inverse transform for $-({q^{\|}}^2/8\mathrm{i}\pi^3\theta)\widetilde{\mathcal{K}}_{ki}^{-1}(\mathbf{q}^{\|})$. The smeared magnetization in real space consists of two parts:

$$M_i^s\left(\mathbf{r}^{\|}, z\right) = \mathcal{M}_i^s\left(\mathbf{r}^{\|}, z\right) - \int \left(\mathcal{K}_{ij}^s\right)^{-1}\left(\left|\mathbf{r}^{\|} - \mathbf{r}^{\|\prime}\right|\right) F_j\left(\mathbf{r}^{\|\prime}, z\right) \mathrm{d}\mathbf{r}^{\|\prime} \tag{19}$$

where the first term $\mathcal{M}_i^s(\mathbf{r}^{\|}, z)$ is the undetermined part of the smeared magnetization, while the second term is the partial information which may be extracted. We find that the smeared inverse kernel has the form

$$\mathcal{K}_{ij}^{s\;-1} = \frac{1}{16\pi^2\mu} \frac{1}{\left|\mathbf{r}^{\|} - \mathbf{r}^{\|\prime}\right|} \times \left[2\left(\hat{\mathbf{r}}^{\|} \times \hat{\mathbf{z}}\right)_i \left(\hat{\mathbf{r}}^{\|} \times \hat{\mathbf{z}}\right)_j - \hat{z}_i\hat{z}_j - \hat{z}_i\hat{x}_j^{\|}\right]. \tag{20}$$

5 Ambiguity in smeared magnetization

In this section, we will characterize the undetermined part of the smeared magnetization. This is readily achieved using the Fourier transform of it, $\widetilde{\mathcal{M}}^s(\mathbf{q}^{\|}, z)$. By definition, we can write $\mathbf{q}^+ \cdot \widetilde{\mathcal{M}}^s(\mathbf{q}^{\|}, z) = 0$, which implies that the vector $\mathbf{q}^+$ is perpendicular to $\widetilde{\mathcal{M}}^s(\mathbf{q}^{\|}, z)$. For this reason, the most economical representation of $\widetilde{\mathcal{M}}^s(\mathbf{q}^{\|}, z)$ is given by the cross product $\mathrm{i}(\mathbf{q}^+ \times \widetilde{\mathbf{A}}(\mathbf{q}^{\|}, z))$, where $\widetilde{\mathbf{A}}(\mathbf{q}^{\|}, z)$ stands for the Fourier transform of an arbitrary vector field. Note that the factor i is put for the sake of convenience. Let us now write out this expression:

$$\widetilde{\mathcal{M}}^s\left(\mathbf{q}^{\|}, z\right) = \mathrm{i}\mathbf{q}^{\|} \times \widetilde{\mathbf{A}}\left(\mathbf{q}^{\|}, z\right) - q^{\|}\left(\hat{\mathbf{z}} \times \widetilde{\mathbf{A}}\left(\mathbf{q}^{\|}, z\right)\right) \tag{21}$$

where we have used the definition of $\mathbf{q}^+$. It should be noted that we are determining the ambiguity in the smeared magnetization. Hence, the vector $\widetilde{\mathbf{A}}(\mathbf{q}^{\|}, z)$ should be interpreted as

$$\widetilde{\mathbf{A}}\left(\mathbf{q}^{\|}, z\right) = \widetilde{\mathbf{A}}\left(\mathbf{q}^{\|}\right) \mathrm{e}^{-q^{\|} z}. \tag{22}$$

In this case we obtain for the ambiguous part

$$\widetilde{\mathcal{M}}^s\left(\mathbf{q}^{\|}, z\right) = \left[\mathrm{i}\mathbf{q}^{\|} + \frac{\partial}{\partial z}\hat{\mathbf{z}}\right] \times \widetilde{\mathbf{A}}\left(\mathbf{q}^{\|}\right) \mathrm{e}^{-q^{\|} z}. \tag{23}$$

If we inverse Fourier transform this expression, we can find the form of ambiguity in real space

$$\mathcal{M}^s\left(\mathbf{r}^{\|}, z\right) = \nabla \times \mathbf{A}\left(\mathbf{r}^{\|}, z\right). \tag{24}$$

Thus, the ambiguity in the smeared magnetization is equal to $\nabla \times \mathbf{A}(\mathbf{r}^{\|}, z)$, given the special form of the vector field.

It is interesting to indicate a similarity with the solution of Maxwell's equation, $\nabla \cdot \mathbf{D}(\mathbf{r}) = 4\pi\rho$, which when Fourier transformed yields $\mathrm{i}\mathbf{q}\widetilde{\mathbf{D}}(\mathbf{q}) = 4\pi\widetilde{\rho}(\mathbf{q})$. It is clear that we can express $\widetilde{\mathbf{D}}(\mathbf{q})$ as follows.

$$\widetilde{\mathbf{D}}(\mathbf{q}) = \mathrm{i}\mathbf{q}\widetilde{\phi}(\mathbf{q}) + \mathrm{i}\mathbf{q} \times \widetilde{\mathbf{A}}(\mathbf{q}) \tag{25}$$

where $\widetilde{\phi}(\mathbf{q}) = -4\pi\widetilde{\rho}(\mathbf{q})/q^2$. The ambiguity in $\widetilde{\mathbf{D}}(\mathbf{q})$ is represented by the second term containing the vector field $\mathbf{A}$. On inverse Fourier transformation one can obtain $\mathbf{D}(\mathbf{r}) = \nabla\phi(\mathbf{r}) + \nabla \times \mathbf{A}(\mathbf{r})$, where the second term is the undetermined part.

6 Conclusions

In this letter, we have seen that a restricted amount of information about the magnetization distribution of a thin ferromagnetic film may be deduced in a model independent manner.

The situation is somewhat similar to the deduction of the electric field from a charge distribution. In this case, the transverse parts of the electric field are not determined. In our example, the physical interpretation of the undetermined part of the magnetization is not so obvious. We have represented the ambiguous part of the magnetization as $\mathrm{i}(\mathbf{q}^+ \times \widetilde{\mathbf{A}}(\mathbf{q}^{\|}, z))$ which has the Fourier transform of $\nabla \times \mathbf{A}(\mathbf{r})$, assuming that we have $\widetilde{\mathbf{A}}(\mathbf{q}^{\|}, z) = \widetilde{\mathbf{A}}(\mathbf{q}^{\|})\mathrm{e}^{-q^{\|} z}$. Thus, the ambiguity is equivalent to $\nabla \times \mathbf{A}(\mathbf{r}^{\|}, z)$.

We may regard the origin of the ambiguity as being essentially the same as that of the electromagnetic case. The field outside the thin film, caused by the film, may be taken to be $\mathbf{H}(\mathbf{r})$. Then, according to Maxwell's equation we can write

$$\nabla \cdot \mathbf{B} = 0 \rightarrow \nabla \cdot \mathbf{H} = -4\pi\nabla \cdot \mathbf{M} = \rho_m.$$

We see that the transverse part of $\mathbf{M}$ cannot affect $\mathbf{H}$, and hence cannot be measured by the effect of $\mathbf{H}$ on the tip.

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